Name (IN CAPITALS): **Version #1**

Instructor: Dora The Explorer

Math 10550/10350 Exam 1
Feb. 12, 2026.

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off (and Put Away) all cellphones, smartwatches and electronic devices.
- Calculators are **not** allowed.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all 18 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-16.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(●)	(b)	(c)	(d)	(e)
2.	(●)	(b)	(c)	(d)	(e)
3.	(●)	(b)	(c)	(d)	(e)
4.	(●)	(b)	(c)	(d)	(e)
5.	(●)	(b)	(c)	(d)	(e)
6.	(●)	(b)	(c)	(d)	(e)
7.	(●)	(b)	(c)	(d)	(e)
8.	(●)	(b)	(c)	(d)	(e)
9.	(●)	(b)	(c)	(d)	(e)
10.	(●)	(b)	(c)	(d)	(e)
11.	(●)	(b)	(c)	(d)	(e)
12.	(●)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice ______

13. ______

14. ______

15. ______

16. ______

Total ______

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1.	(a)	(b)	(c)	(d)	(e)
2	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice ________________

13. ________________

14. ________________

15. ________________

16. ________________

Total ________________

Multiple Choice

1.(7pts) The following table shows the position, $s(t)$, at time t, of a particle moving on an axis, where t is measured in seconds (s) and distance is measured in meters (m).

t	0	1	2	3	4	5	6	7	8	9	10
$s(t)$	1	-3	1	0	1	-1	-4	4	3	2	1

Which of the following gives the average velocity of the particle, between $t = 2$ and $t = 5$ seconds?

Solution:

$$\text{avg. velocity} = \frac{s(5) - s(2)}{5 - 2} = \frac{-1 - 1}{3} = -\frac{2}{3} \text{ m/s}$$

(a) $-\frac{2}{3}$ m/s (b) $-\frac{2}{5}$ m/s (c) -2 m/s (d) $\frac{1}{2}$ m/s (e) 0 m/s

2.(7pts) Let $f(x) = 2x$ and $g(x) = \cos(x)$. Which of the following is equal to

$$g\left(f\left(\frac{\pi}{2}\right)\right)?$$

Solution:

$$g\left(f\left(\frac{\pi}{2}\right)\right) = g\left(2 \cdot \frac{\pi}{2}\right) = g(\pi) = \cos(\pi) = -1$$

(a) -1 (b) 1 (c) -2 (d) 0 (e) 2

3.(7pts) Evaluate the following limit:

$$\lim_{x\to 3^+} \frac{\sqrt{x^2+7}-4}{x-3}.$$

Solution: First, rewrite the function inside the limit using conjugates.

$$\begin{aligned}\frac{\sqrt{x^2+7}-4}{x-3} &= \frac{\sqrt{x^2+7}-4}{x-3}\cdot\frac{\sqrt{x^2+7}+4}{\sqrt{x^2+7}+4} = \frac{(x^2+7)-16}{(x-3)(\sqrt{x^2+7}+4)}\\ &= \frac{x^2-9}{(x-3)(\sqrt{x^2+7}+4)} = \frac{(x-3)(x+3)}{(x-3)(\sqrt{x^2+7}+4)}\\ &= \frac{x+3}{\sqrt{x^2+7}+4}\end{aligned}$$

Now, evaluate the limit

$$\lim_{x\to 3^+} \frac{\sqrt{x^2+7}-4}{x-3} = \lim_{x\to 3^+} \frac{x+3}{\sqrt{x^2+7}+4} = \frac{6}{\sqrt{16}+4} = \frac{6}{8} = \frac{3}{4}$$

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{3}{2}$ (d) $+\infty$ (e) $\frac{1}{2}$

4.(7pts) Compute

$$\lim_{x\to 2^-} \frac{x^2 - x - 2}{x^2 - 4x + 4}.$$

Solution:

$$\lim_{x\to 2^-} \frac{x^2 - x - 2}{x^2 - 4x + 4} = \lim_{x\to 2^-} \frac{(x-2)(x+1)}{(x-2)(x-2)} = \lim_{x\to 2^-} \frac{x+1}{x-2}$$
$$= \lim_{x\to 2^-} (x+1) \cdot \lim_{x\to 2^-} \frac{1}{x-2} = 3 \cdot (-\infty) = -\infty$$

(a) $-\infty$

(b) 0

(c) $+\infty$

(d) 1

(e) Does not exist and is not $+\infty$ or $-\infty$

5.(7pts) Evaluate $\lim_{x\to+\infty} \dfrac{\sqrt{4x^2+5}}{x+1}$.

Solution:

$$\begin{aligned}\lim_{x\to+\infty} \frac{\sqrt{4x^2+5}}{x+1} &= \lim_{x\to+\infty} \frac{\sqrt{x^2(4+\frac{5}{x^2})}}{x(1+\frac{1}{x})} = \lim_{x\to+\infty} \frac{|x|\sqrt{4+\frac{5}{x^2}}}{x(1+\frac{1}{x})} \\ &= \lim_{x\to+\infty} \frac{x\sqrt{4+\frac{5}{x^2}}}{x(1+\frac{1}{x})} = \lim_{x\to+\infty} \frac{\sqrt{4+\frac{5}{x^2}}}{1+\frac{1}{x}} = \frac{\sqrt{4+0}}{1+0} = \frac{2}{1} = 2\end{aligned}$$

(a) 2 (b) -2 (c) 4 (d) $+\infty$ (e) -4

6.(7pts) Consider the piecewise defined function:

$$f(x) = \begin{cases} 2 + x - x^3 & \text{if } -\infty < x < -1 \\ -2x & \text{if } -1 \le x < 1 \\ x^2 - 1 & \text{if } 1 \le x < +\infty \end{cases}$$

Which of the following statements is true?

Solution:

(a) True. First, check that f is continuous at $x = -1$.

$$f(-1) = -2(-1) = 2$$
$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (2 + x - x^3) = 2 + (-1) - (-1)^3 = 2$$
$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} -2x = -2(-1) = 2$$

Because the limits as x approaches -1 from the left and from the right agree, $\lim_{x\to -1} f(x)$ exists. Since $\lim_{x\to -1} f(x) = 2 = f(-1)$, f is continuous at $x = -1$. Next, check if f is differentiable at $x = -1$. Using the definition of f on the interval $-\infty < x < -1$, compute the derivative according to the power rule. Then, $f'(x) = (2 + x - x^3)' = 1 - 3x^2$ and $f'(-1) = 1 - 3(1) = -2$. On the other hand, using the definition of f on the interval $-1 \le x < 1$, compute the derivative according to the power rule. Then, $f'(x) = (-2x)' = -2$, and $f'(-1) = -2$. Because the derivatives on each side of the piecewise function are equal, then f is differentiable at $x = -1$, and $f'(-1) = -2$.

(b) False. See explanation for (a).

(c) False. From (d), f is not continuous at $x = 1$, so f cannot be differentiable at this point.

(d) False. The limit at $x = 1$ does not exist since

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} -2x = -2 \ne \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2 - 1 = 0,$$

so f is not continuous at $x = 1$.

(e) False. $f(1) = 0$.

(a) $f'(-1) = -2$.

(b) $f(x)$ is not differentiable at $x = -1$.

(c) $f'(1) = -2$.

(d) $f(x)$ is continuous at $x = 1$.

(e) $f(1) = 1$.

7.(7pts) Let

$$f(x) = \sqrt{x} - \frac{4}{\sqrt{x}}$$

Which of the following gives the equation of the tangent line to the graph of $y = f(x)$ at $x = 4$?

Solution: To find the slope of the tangent line to the graph at $x = 4$, compute $f'(4)$.

$$\begin{aligned} f'(4) &= \lim_{h\to 0} \frac{f(4+h) - f(4)}{h} = \lim_{h\to 0} \frac{\sqrt{4+h} - \frac{4}{\sqrt{4+h}} - \left(\sqrt{4} - \frac{4}{\sqrt{4}}\right)}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{4+h} - \frac{4}{\sqrt{4+h}}}{h} = \lim_{h\to 0} \frac{(4+h) - 4}{h\sqrt{4+h}} = \lim_{h\to 0} \frac{h}{h\sqrt{4+h}} \\ &= \lim_{h\to 0} \frac{1}{\sqrt{4+h}} = \frac{1}{\sqrt{4}} = \frac{1}{2} \end{aligned}$$

Now, the equation of the tangent line to the graph of f at $x = 4$ is $y = \frac{1}{2}x + b$, with y-intercept b. To solve for b, use a point on the line, namely $(4, f(4)) = (4, 0)$, and substitute into the equation. Then, $0 = \frac{1}{2} \cdot 4 + b$, so $b = -2$, and the desired equation is $y = \frac{1}{2}x - 2$.

(a) $y = \dfrac{1}{2}x - 2$

(b) $y = \dfrac{1}{8}x - \dfrac{1}{2}$

(c) $y = \dfrac{1}{8}x + 4$

(d) $y = \dfrac{1}{2}x + 4$

(e) $y = x - 4$

8.(7pts) What are the asymptotes, both vertical and horizontal, for the graph of

$$y = \frac{2x^2 - 2}{(x+1)(2x-6)}?$$

Solution:

$$y = \frac{2(x^2-1)}{2(x+1)(x-3)} = \frac{(x+1)(x-1)}{(x+1)(x-3)} = \frac{x-1}{x-3}.$$

To find the vertical asymptotes, check the limits at x values where y is undefined.

$$\lim_{x\to 3^-} \frac{x-1}{x-3} = -\infty, \text{ and } \lim_{x\to 3^+} \frac{x-1}{x-3} = \infty.$$

Since these limits are $\pm\infty$, then the graph of y has a vertical asymptote at $x = 3$.
To find the horizontal asymptotes, check the limits as x approaches $\pm\infty$.

$$\lim_{x\to\pm\infty} \frac{x-1}{x-3} = \lim_{x\to\pm\infty} \frac{x(1-\frac{1}{x})}{x(1-\frac{3}{x})} = \lim_{x\to\pm\infty} \frac{1-\frac{1}{x}}{1-\frac{3}{x}} = 1.$$

Then the graph of y has a horizontal asymptote at $y = 1$.

(a) $y = 1, \quad x = 3$

(b) $y = 2, \quad x = 3$ and $x = -1$

(c) $y = 2, \quad x = 3$

(d) $y = 1, \quad x = -1, \quad x = -3$

(e) $y = 2, \quad x = 1, \quad x = -3$

9.(7pts) For what constant a is the function f given by

$$f(x) = \begin{cases} \dfrac{x^2 + x - 2}{x - 1} & x < 1 \\ \\ ax + 1 & x \geq 1 \end{cases}$$

continuous at $x = 1$?

Solution:

$$f(1) = a + 1$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x^2 + x - 2}{x - 1} = \lim_{x \to 1^-} \frac{(x-1)(x+2)}{x - 1} = \lim_{x \to 1^-} (x + 2) = 3$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (ax + 1) = a + 1$$

In order for f to be continuous at $x = 1$, these two limits must be equal, so $3 = a + 1$ and $a = 2$.

(a) $a = 2$

(b) $a = 3$

(c) $a = 4$

(d) $a = 0$

(e) No such value of a exists

10.(7pts) If $f(x)$ and $g(x)$ are functions for which $\lim_{x\to 0} f(x) = 4$ and $\lim_{x\to 0} g(x) = 9$, what is

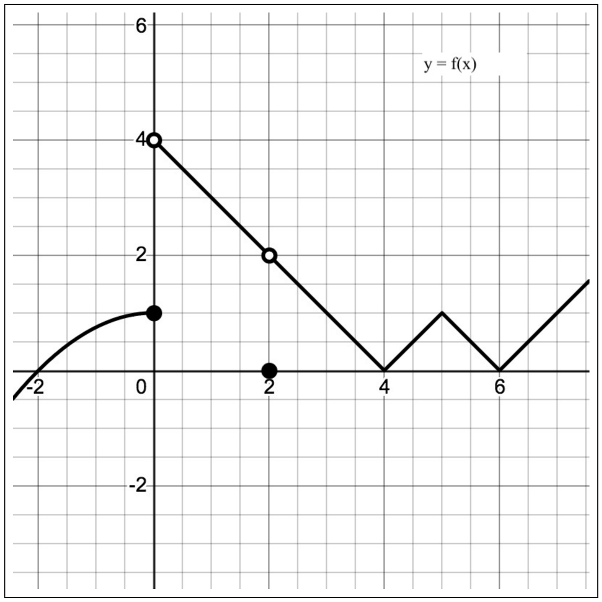
$$\lim_{x\to 0}\left(\frac{f(x)}{g(x)} - \frac{e^x}{\sqrt{g(x)}}\right).$$

Solution:

$$\lim_{x\to 0}\left(\frac{f(x)}{g(x)} - \frac{e^x}{\sqrt{g(x)}}\right) = \lim_{x\to 0}\frac{f(x)}{g(x)} - \lim_{x\to 0}\frac{e^x}{\sqrt{g(x)}} = \frac{\lim_{x\to 0} f(x)}{\lim_{x\to 0} g(x)} - \frac{\lim_{x\to 0} e^x}{\lim_{x\to 0}\sqrt{g(x)}}$$
$$= \frac{\lim_{x\to 0} f(x)}{\lim_{x\to 0} g(x)} - \frac{\lim_{x\to 0} e^x}{\sqrt{\lim_{x\to 0} g(x)}} = \frac{4}{9} - \frac{1}{\sqrt{9}} = \frac{4}{9} - \frac{1}{3}$$
$$= \frac{4}{9} - \frac{3}{9} = \frac{1}{9}.$$

(a) $\frac{1}{9}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $+\infty$ (e) $-\infty$

11.(7pts) The graph of the function $y = f(x)$ is shown below.



Which of the following statements is true?

Solution:

(a) True. Disregard all parts of the graph which are outside the interval $[-2, 0]$. For all x-values in the interval $(-2, 0)$, $f(x)$ is clearly continuous. At the endpoints of the interval, since $\lim_{x\to -2^+} f(x) = f(-2)$ and $\lim_{x\to 0^-} f(x) = f(0)$, $f(x)$ is continuous.

(b) False. $fx)$ is not continuous on $[0, 2)$ since $\lim_{x\to 0^+} f(x) = 4 \neq f(0) = 1$.

(c) False. At $x = 4$, $f(x)$ is not differentiable.

(d) False. $\lim_{x\to 2} f(x) = 2$.

(e) False. The discontinuity at $x = 0$ is not removable because the limits on the left and right sides of 0 do not agree.

(a) $f(x)$ is continuous on the interval $[-2, 0]$

(b) $f(x)$ is continuous on the interval $[0, 2)$

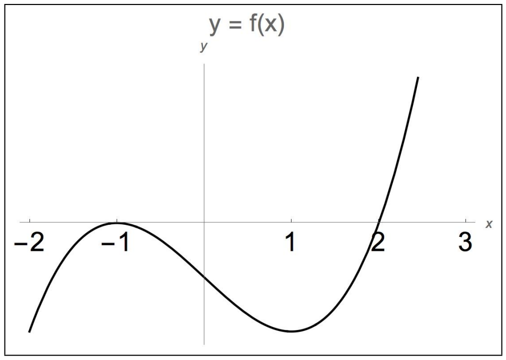
(c) $f'(4) = 0$

(d) $\lim\limits_{x\to 2} f(x)$ does not exist

(e) $f(x)$ has a removable discontinuity at $x = 0$

Initials: ____________

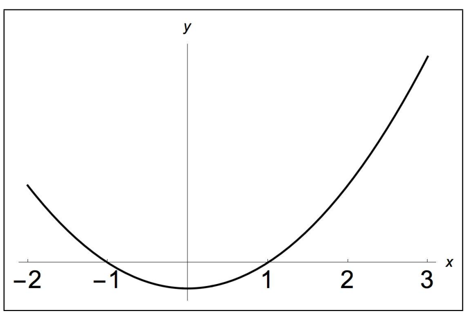
12.(7pts) The graph of $f(x)$ is shown below:



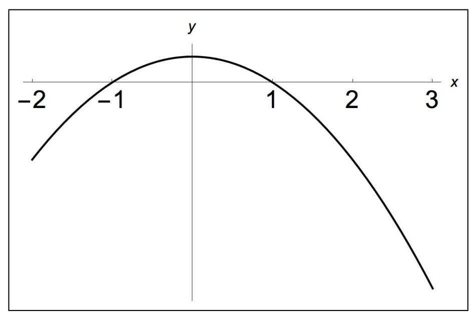
Which of the following is the graph of $f'(x)$?

Solution: The derivative of f is 0 wherever f has a local maximum or a local minimum, so at $x = -1, 1$, $f'(x) = 0$. In addition, f is increasing for x-values to the left of -1, so has positive derivative here, f is decreasing for x-values between -1 and 1, so has negative derivative here, and f is again increasing for x-values to the right of 2, so has positive derivative here. The only graph that meets all of these conditions is graph (a).

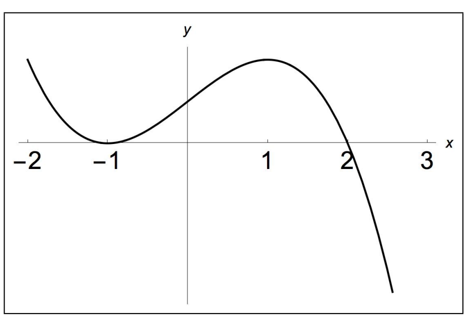
(a)



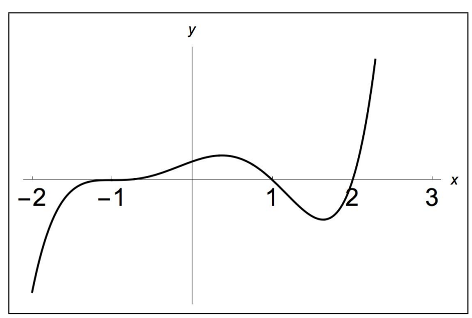
(b)



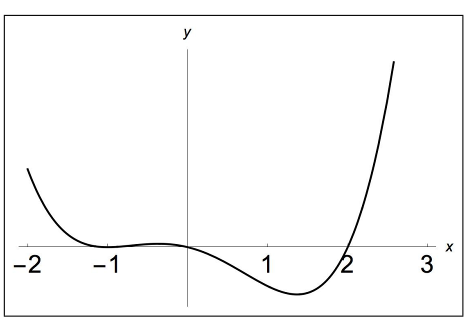
(c)



(d)



(e)



Partial Credit

For full credit on questions 13 and 14, make sure you justify all of your answers.

13.(12pts) (a) Suppose $f(x)$ is a function for which we do not have a formula, but we know the following to be true:

- The domain of $f(x)$ is the set of all real numbers and
- $x^2 - 4x + 1 \leq f(x) \leq 2x^2 - 8x + 5$ for all values of x.

Use the squeeze theorem to find $\lim\limits_{x \to 2} f(x)$, or explain why the squeeze theorem doesn't help here.

Solution: Since the domain of $f(x)$ is all real numbers, then we know that f is defined everywhere, so can take its limit as x approaches 2. Take the limit as x approaches 2 of the inequality given in the second bullet point.

$$\lim_{x \to 2}(x^2 - 4x + 1) \leq \lim_{x \to 2} f(x) \leq \lim_{x \to 2}(2x^2 - 8x + 5).$$

Compute the limits on each side of the inequality.

$$\lim_{x \to 2}(x^2 - 4x + 1) = (2)^2 - 4(2) + 1 = 4 - 8 + 1 = -3$$
$$\lim_{x \to 2}(2x^2 - 8x + 5) = 2(2)^2 - 8(2) + 5 = 8 - 16 + 5 = -3.$$

Plug this into the inequality,

$$-3 \leq \lim_{x \to 2} f(x) \leq -3.$$

By the squeeze theorem, $\lim_{x \to 2} f(x) = -3$.

(b) Consider the function

$$g(x) = x^3 + 10x - 5.$$

Explain why $g(c) = 0$ for some number c in the interval $[0, 1]$.

(**Note:** Do not try to actually find c.)

Solution: The graph of $g(x)$ is continuous everywhere. $g(0) = -5$ is negative and $g(1) = 1 + 10 - 5 = 6$ is positive, then by the intermediate value theorem, $f(c)$ must equal 0 for some c in the interval $[0, 1]$.

14.(12pts) (a) Find the derivative of

$$f(x) = \frac{1}{x+1}$$

using the limit definition of the derivative.
(no points here for using a differentiation formula)

Solution:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = \lim_{h\to 0} \frac{(x+1)-(x+h+1)}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{-h}{h(x+h+1)(x+1)} = \lim_{h\to 0} \frac{-1}{(x+h+1)(x+1)} = \frac{-1}{(x+1)^2}.$$

(b) Use your answer from part (a) to find the equation of the tangent line to the graph of $y = f(x)$ at $x = 1$.

Solution: The slope of the tangent line to the graph of f at $x = 1$ is $f'(1) = -\frac{1}{4}$. Use the point $(1, f(1)) = (1, \frac{1}{2})$ to find the y-intercept of the tangent line in the equation $y = -\frac{1}{4}x + b$. Then, $\frac{1}{2} = -\frac{1}{4} \cdot 1 + b$, so $b = \frac{3}{4}$. The equation of the tangent line is $y = -\frac{1}{4}x + \frac{3}{4}$.

True-False.

15.(4pts) Please circle "TRUE" if you think the statement is true, and circle "FALSE" if you think the statement is False.

(a)(1 pt. No Partial credit) The graph of the function $f(x) = \dfrac{(x+1)(x-1)}{x-1}$ has a vertical asymptote at $x = 1$.

Solution: $f(x) = \frac{(x+1)(x-1)}{x-1} = x + 1$ has a hole at $x = 1$.

TRUE **<u>FALSE</u>**

(b)(1 pt. No Partial credit) $\lim\limits_{x\to 0^+} \ln(x) = 0$

Solution: $\lim_{x\to 0^+} \ln(x) = -\infty$.

TRUE **<u>FALSE</u>**

(c)(1 pt. No Partial credit)

If $f(x) = 2\sqrt{x}$, then $f''(1) = -\dfrac{1}{2}$.

Solution: Rewrite the function $f(x) = 2(x)^{\frac{1}{2}}$, and use the power rule to get $f'(x) = x^{-\frac{1}{2}}$. Use the power rule again to get $f''(x) = -\frac{1}{2}(x)^{-\frac{3}{2}}$. Then $f''(1) = -\frac{1}{2}(1)^{-\frac{3}{2}} = -\frac{1}{2}$.

<u>TRUE</u> FALSE

(d)(1 pt. No Partial credit) $\lim\limits_{x\to+\infty} e^x = +\infty$

Solution: Consult the graph of e^x to see that this is true.

<u>TRUE</u> FALSE

16.(2pts) You will be awarded these two points if you write your name in CAPITALS on the front page and you mark your answers on the front page with an X through your answer choice like so: (a) (<u>not</u> an O around your answer choice) .

ROUGH WORK